

Calcul des déformations des ressorts hélicoïdaux

Forces et couples concentrés - Modèle encastré - libre

Torseur des forces de cohésion

$$\mathbf{r}_F(r_F, \chi_F, \psi_F) := r_F \cdot \begin{pmatrix} \sin(\chi_F) \cdot \cos(\psi_F) \\ \sin(\chi_F) \cdot \sin(\psi_F) \\ \cos(\chi_F) \end{pmatrix} \quad \mathbf{r}_S(\alpha') := \begin{pmatrix} x_0(\alpha') \\ y_0(\alpha') \\ z_0(\alpha') \end{pmatrix} \quad \mathbf{r}_V(\alpha) := \begin{pmatrix} x_0(\alpha) \\ y_0(\alpha) \\ z_0(\alpha) \end{pmatrix}$$

Forces et couples concentrés en ψ_F

$$\mathbf{R}_c(\alpha') := \mathbf{F}$$

$$\mathbf{M}_c(\alpha') := [\mathbf{C} + (\mathbf{r}_F(r_F, \chi_F, \psi_F) - \mathbf{r}_S(\alpha')) \times \mathbf{F}]$$

Sollicitations

$$\mathbf{e}'_1(\alpha', \beta) := (-\cos(\beta) \cdot \sin(\alpha') \quad \cos(\beta) \cdot \cos(\alpha') \quad \sin(\beta))^T \quad \mathbf{e}'_2(\alpha', \beta) := (-\cos(\alpha') \quad -\sin(\alpha') \quad 0)^T$$

$$\mathbf{e}'_3(\alpha', \beta) := (\sin(\beta) \cdot \sin(\alpha') \quad -\sin(\beta) \cdot \cos(\alpha') \quad \cos(\beta))^T$$

Traction-compression $N_c(\alpha') := \mathbf{R}_c(\alpha') \cdot \mathbf{e}'_1(\alpha', \beta)$

Effort tranchant $Q_2(\alpha') := \mathbf{R}_c(\alpha') \cdot \mathbf{e}'_2(\alpha', \beta)$

$$Q_3(\alpha') := \mathbf{R}_c(\alpha') \cdot \mathbf{e}'_3(\alpha', \beta)$$

Moment de torsion $M_t(\alpha') := \mathbf{M}_c(\alpha') \cdot \mathbf{e}'_1(\alpha', \beta)$

Moments de flexion $M_{f2}(\alpha') := \mathbf{M}_c(\alpha') \cdot \mathbf{e}'_2(\alpha', \beta)$

$$M_{f3}(\alpha') := \mathbf{M}_c(\alpha') \cdot \mathbf{e}'_3(\alpha', \beta)$$

Contraintes

k = limite d'élasticité en traction / limite d'élasticité en compression

M=1 point intérieur de la section droite sur l'axe Oe'_2 M=3 point extérieur de la section droite sur l'axe Oe'_2

M=2 point inférieur de la section droite sur l'axe Oe'_3 M=4 point supérieur de la section droite sur l'axe Oe'_3

$$\tau_{Mt}(\alpha', M) := \frac{M_t(\alpha')}{W_t} \cdot [(M=1) - 1 \cdot (M=3)] + \frac{M_t(\alpha')}{W'_t} \cdot [(M=4) - 1 \cdot (M=2)]$$

$$\tau_Q(\alpha', M) := \frac{Q_2(\alpha')}{S} \cdot [(M=2) + (M=4)] + \frac{Q_3(\alpha')}{S} \cdot [(M=1) + (M=3)] \quad (\text{approximation})$$

$$\sigma_{Mf}(\alpha', M) := \frac{M_{f3}(\alpha')}{W_{f3}} \cdot [(M=3) - 1 \cdot (M=1)] + \frac{M_{f2}(\alpha')}{W_{f2}} \cdot [(M=2) - 1 \cdot (M=4)]$$

$$\sigma_N(\alpha') := \frac{N_c(\alpha')}{S} \quad \sigma_M(\alpha', M) := \sigma_{Mf}(\alpha', M) + \sigma_N(\alpha') \quad \tau_M(\alpha', M) := \tau_{Mt}(\alpha', M) + \tau_Q(\alpha', M)$$

$$\sigma_{\text{equiv}}(k, \alpha', M) := \frac{1-k}{2} \cdot |\sigma_M(\alpha', M)| + \frac{1+k}{2} \cdot \sqrt{\sigma_M(\alpha', M)^2 + 4 \cdot \tau_M(\alpha', M)^2}$$

Calcul des déplacements par les intégrales de Mohr

Calcul des déplacements linéiques d'un point du ressort

Force unitaire virtuelle $\mathbf{v}(\lambda, \gamma) := (\cos(\lambda) \cdot \sin(\gamma) \quad \sin(\lambda) \cdot \sin(\gamma) \quad \cos(\gamma))^T$

Sollicitations dues à la force unitaire

$$\mathbf{M}_v(\alpha, \lambda, \gamma, \alpha') := \left[(\mathbf{r}_v(\alpha) - \mathbf{r}_s(\alpha')) \times \mathbf{v}(\lambda, \gamma) \right] \cdot (\alpha' < \alpha) \quad M_{tv}(\alpha, \lambda, \gamma, \alpha') := \mathbf{M}_v(\alpha, \lambda, \gamma, \alpha') \cdot \mathbf{e}_1(\alpha', \beta)$$

$$M_{fv2}(\alpha, \lambda, \gamma, \alpha') := \mathbf{M}_v(\alpha, \lambda, \gamma, \alpha') \cdot \mathbf{e}_2(\alpha', \beta) \quad M_{fv3}(\alpha, \lambda, \gamma, \alpha') := \mathbf{M}_v(\alpha, \lambda, \gamma, \alpha') \cdot \mathbf{e}_3(\alpha', \beta)$$

Déplacement dans la direction de \mathbf{v}

$$\delta_{tv}(\alpha, \lambda, \gamma) := \frac{1}{G \cdot J_t} \cdot \int_0^\alpha \mathbf{M}_t(\alpha') \cdot M_{tv}(\alpha, \lambda, \gamma, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\delta_{fv2}(\alpha, \lambda, \gamma) := \frac{1}{E \cdot I_{22}} \cdot \int_0^\alpha \mathbf{M}_{f2}(\alpha') \cdot M_{fv2}(\alpha, \lambda, \gamma, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\delta_{fv3}(\alpha, \lambda, \gamma) := \frac{1}{E \cdot I_{33}} \cdot \int_0^\alpha \mathbf{M}_{f3}(\alpha') \cdot M_{fv3}(\alpha, \lambda, \gamma, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\delta_v(\alpha, \lambda, \gamma) := \delta_{tv}(\alpha, \lambda, \gamma) + \delta_{fv2}(\alpha, \lambda, \gamma) + \delta_{fv3}(\alpha, \lambda, \gamma)$$

Calcul des déplacements angulaires

Couple unitaire virtuel $\mathbf{cv}(\lambda_c, \gamma_c) := (\cos(\lambda_c) \cdot \sin(\gamma_c) \quad \sin(\lambda_c) \cdot \sin(\gamma_c) \quad \cos(\gamma_c))^T$

Sollicitations dues au couple unitaire

$$\mathbf{M}_{cv}(\alpha, \lambda_c, \gamma_c, \alpha') := \mathbf{cv}(\lambda_c, \gamma_c) \cdot (\alpha' < \alpha)$$

$$M_{tcv}(\alpha, \lambda_c, \gamma_c, \alpha') := \mathbf{M}_{cv}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \mathbf{e}_1(\alpha', \beta)$$

$$M_{fcv2}(\alpha, \lambda_c, \gamma_c, \alpha') := \mathbf{M}_{cv}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \mathbf{e}_2(\alpha', \beta) \quad M_{fcv3}(\alpha, \lambda_c, \gamma_c, \alpha') := \mathbf{M}_{cv}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \mathbf{e}_3(\alpha', \beta)$$

Déplacement angulaire autour de l'axe défini par \mathbf{cv}

$$\theta_{tcv}(\alpha, \lambda_c, \gamma_c) := \frac{1}{G \cdot J_t} \cdot \int_0^\alpha \mathbf{M}_t(\alpha') \cdot M_{tcv}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\theta_{fcv2}(\alpha, \lambda_c, \gamma_c) := \frac{1}{E \cdot I_{22}} \cdot \int_0^\alpha \mathbf{M}_{f2}(\alpha') \cdot M_{fcv2}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\theta_{fcv3}(\alpha, \lambda_c, \gamma_c) := \frac{1}{E \cdot I_{33}} \cdot \int_0^\alpha \mathbf{M}_{f3}(\alpha') \cdot M_{fcv3}(\alpha, \lambda_c, \gamma_c, \alpha') \cdot \frac{R}{\cos(\beta)} d\alpha'$$

$$\theta_{cv}(\alpha, \lambda_c, \gamma_c) := \theta_{tcv}(\alpha, \lambda_c, \gamma_c) + \theta_{fcv2}(\alpha, \lambda_c, \gamma_c) + \theta_{fcv3}(\alpha, \lambda_c, \gamma_c)$$